LESSON 4.1c

Polynomial Identities and Patterns

Today you will:

- Learn the special polynomial product patterns
- Practice using English to describe math processes and equations

Core Vocabulary:

- Identity
- Proving an identity
- Pascal’s Triangle, p. 169

Identity

An **identity** is an equality that holds true regardless of the values chosen for its variables.

Proving an Identity

Use logical steps to show that one side of the equation **can** be transformed into the other side of the equation.

COMMON ERROR

In general,

$(a \pm b)^2 \neq a^2 \pm b^2$

and

$(a \pm b)^3 \neq a^3 \pm b^3$.

Core Concept

Special Product Patterns

Sum and Difference

$(a + b)(a - b) = a^2 - b^2$

Example

$(x + 3)(x - 3) = x^2 - 9$

Square of a Binomial

$(a + b)^2 = a^2 + 2ab + b^2$

$(a - b)^2 = a^2 - 2ab + b^2$

Example

$(y + 4)^2 = y^2 + 8y + 16$

$(2t - 5)^2 = 4t^2 - 20t + 25$

Cube of a Binomial

$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

Example

$(z + 3)^3 = z^3 + 9z^2 + 27z + 27$

$(m - 2)^3 = m^3 - 6m^2 + 12m - 8$

a. Prove the polynomial identity for the cube of a binomial representing a sum:

$$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3.$$

b. Use the cube of a binomial in part (a) to calculate 11^3.

SOLUTION

a. Expand and simplify the expression on the left side of the equation.

$$\begin{aligned}(a + b)^3 &= (a + b)(a + b)(a + b)\\ &= (a^2 + 2ab + b^2)(a + b)\\ &= (a^2 + 2ab + b^2)a + (a^2 + 2ab + b^2)b\\ &= a^3 + a^2b + 2a^2b + 2ab^2 + ab^2 + b^3\\ &= a^3 + 3a^2b + 3ab^2 + b^3 \checkmark\end{aligned}$$

The simplified left side equals the right side of the original identity. So, the identity $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$ is true.

b. To calculate 11^3 using the cube of a binomial, note that $11 = 10 + 1$.

$11^3 = (10 + 1)^3$	Write 11 as 10 + 1.
$= 10^3 + 3(10)^2(1) + 3(10)(1)^2 + 1^3$	Cube of a binomial
$= 1000 + 300 + 30 + 1$	Expand.
$= 1331$	Simplify.

$$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

REMEMBER

Power of a Product Property

$(ab)^m = a^m b^m$

a and b are real numbers and m is an integer.

Find each product.

a. $(4n + 5)(4n - 5)$ **b.** $(9y - 2)^2$ **c.** $(ab + 4)^3$

SO

$$(a + b)(a - b) = a^2 - b^2$$

a. $(4n + 5)(4n - 5) = (4n)^2 - 5^2$ Sum and difference

$= 16n^2 - 25$ Simplify.

$$(a - b)^2 = a^2 - 2ab + b^2$$

b. $(9y - 2)^2 = (9y)^2 - 2(9y)(2) + 2^2$ Square of a binomial

$= 81y^2 - 36y + 4$ Simplify.

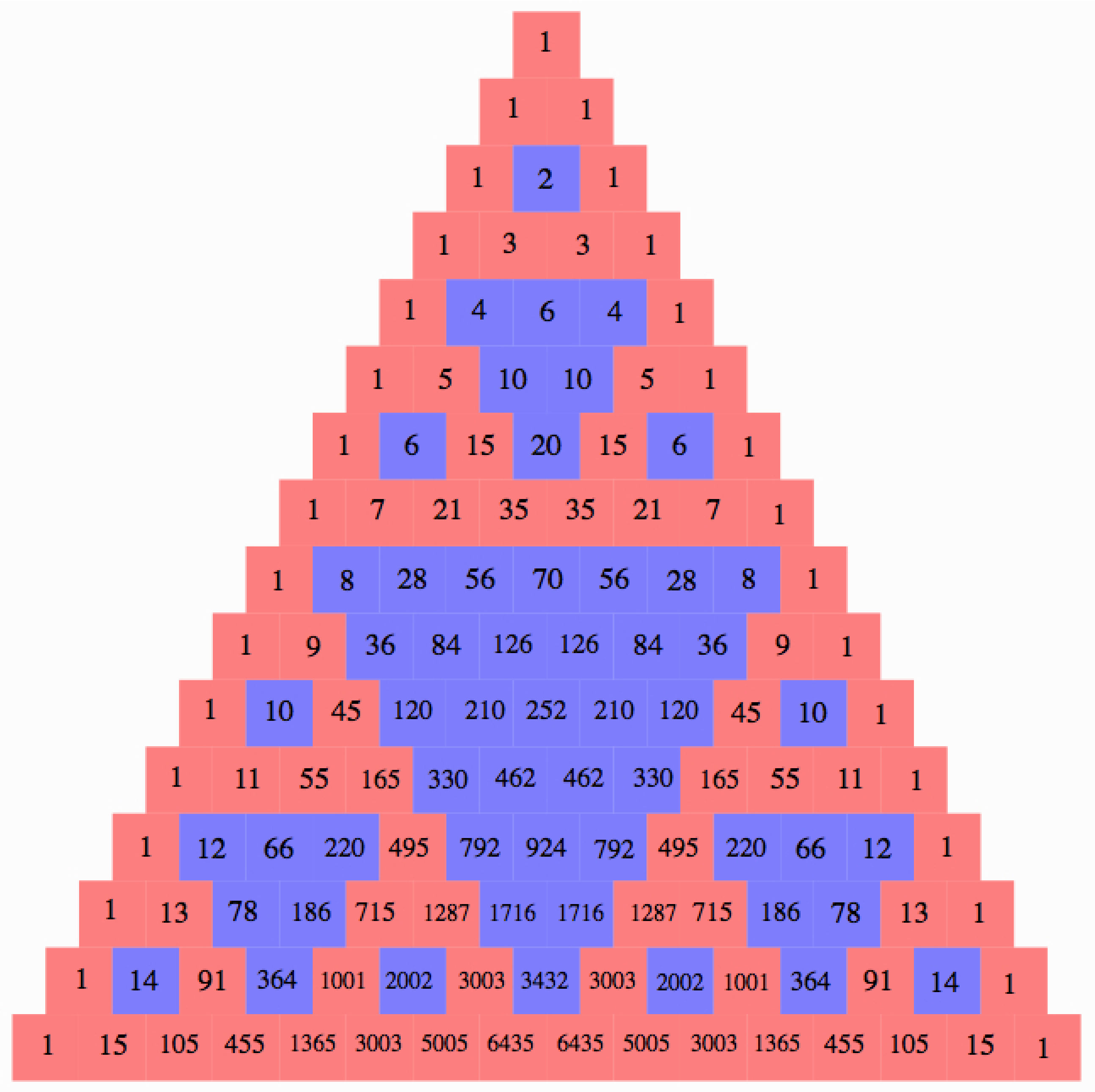
$$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

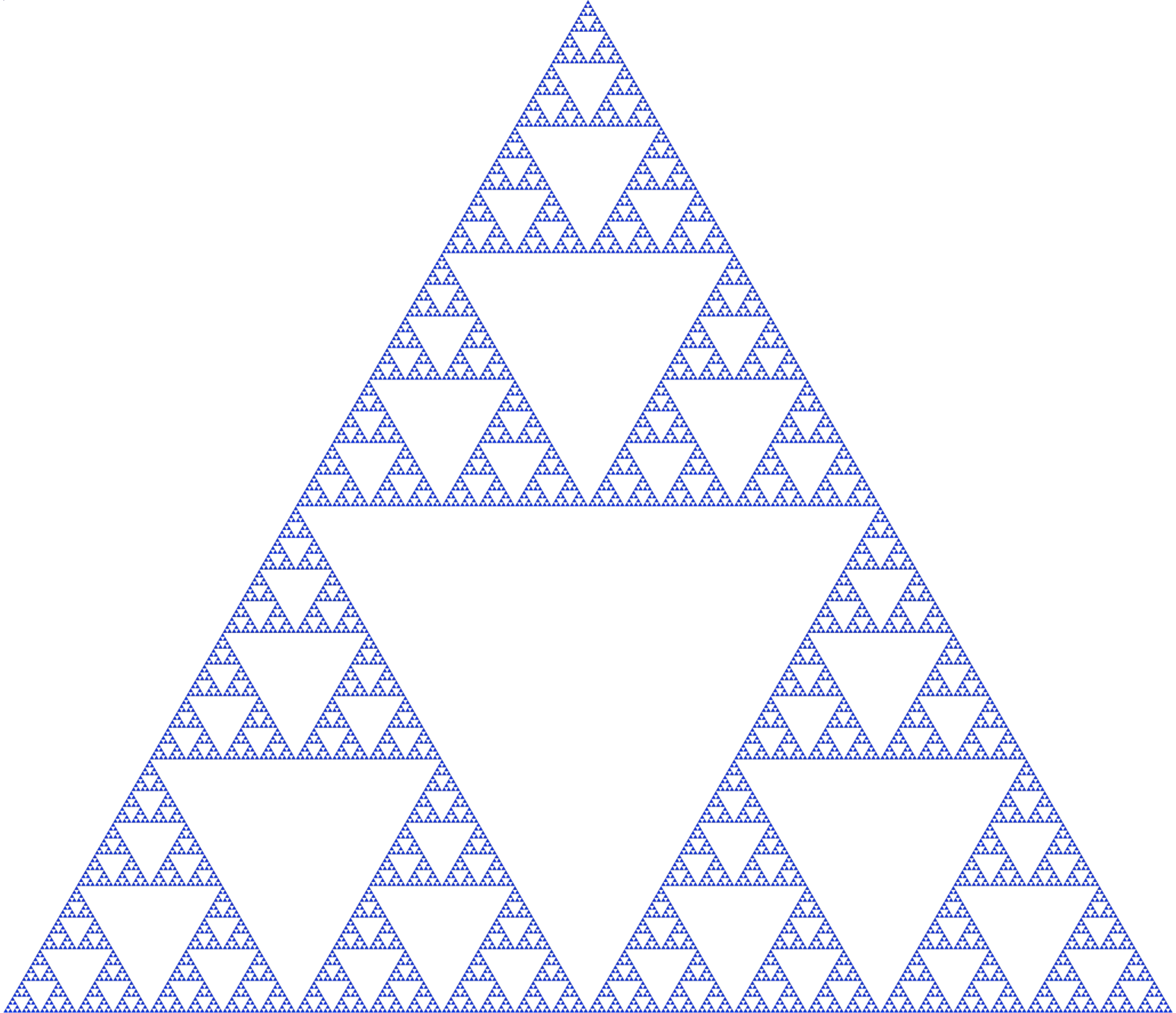
c. $(ab + 4)^3 = (ab)^3 + 3(ab)^2(4) + 3(ab)(4)^2 + 4^3$ Cube of a binomial

$= a^3b^3 + 12a^2b^2 + 48ab + 64$ Simplify.

Pascal’s Triangle

					1					
				1		1				
			1		2		1			
		1		3		3		1		
	1		4		6		4		1	
1		5		10		10		5		1


1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1
1 9 36 84 126 126 84 36 9 1
1 10 45 120 210 252 210 120 45 10 1
1 11 55 165 330 462 462 330 165 55 11 1
1 12 66 220 495 792 924 792 495 220 66 12 1
1 13 78 186 715 1287 1716 1716 1287 715 186 78 13 1
1 14 91 364 1001 2002 3003 3432 3003 2002 1001 364 91 14 1
1 15 105 455 1365 3003 5005 6435 6435 5005 3003 1365 455 105 15 1



Pascal’s Triangle is *FREAKING* cool!

http://jwilson.coe.uga.edu/EMAT6680Su12/Berryman/6690/BerrymanK-Pascals/BerrymanK-Pascals.html

Core Concept

Pascal's Triangle

In Pascal's Triangle, the first and last numbers in each row are 1. Every number other than 1 is the sum of the closest two numbers in the row directly above it. The numbers in Pascal's Triangle are the same numbers that are the coefficients of binomial expansions, as shown in the first six rows.

	n	$(a+b)^n$	Binomial Expansion	Pascal's Triangle
0th row	0	$(a+b)^0 =$	1	1
1st row	1	$(a+b)^1 =$	$1a + 1b$	1 1
2nd row	2	$(a+b)^2 =$	$1a^2 + 2ab + 1b^2$	1 2 1
3rd row	3	$(a+b)^3 =$	$1a^3 + 3a^2b + 3ab^2 + 1b^3$	1 3 3 1
4th row	4	$(a+b)^4 =$	$1a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + 1b^4$	1 4 6 4 1
5th row	5	$(a+b)^5 =$	$1a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + 1b^5$	1 5 10 10 5 1

Use Pascal's Triangle to expand (a) $(x - 2)^5$ and (b) $(3y + 1)^3$.

SOLUTION

a. The coefficients from the fifth row of Pascal's Triangle are 1, 5, 10, 10, 5, and 1.

$$\boldsymbol{a = x}$$
$$\boldsymbol{b = -2}$$

$$1a^5 + 5a^4b^1 + 10a^3b^2 + 10a^2b^3 + 5a^1b^4 + 1b^5$$

$$(x - 2)^5 = 1x^5 + 5x^4(-2) + 10x^3(-2)^2 + 10x^2(-2)^3 + 5x(-2)^4 + 1(-2)^5$$

$$= x^5 - 10x^4 + 40x^3 - 80x^2 + 80x - 32$$

b. The coefficients from the third row of Pascal's Triangle are 1, 3, 3, and 1.

$$\boldsymbol{a = 3y}$$
$$\boldsymbol{b = 1}$$

$$1a^3 + 3a^2b^1 + 3a^1b^2 + 1b^3$$

$$(3y + 1)^3 = 1(3y)^3 + 3(3y)^2(1) + 3(3y)(1)^2 + 1(1)^3$$

$$= 27y^3 + 27y^2 + 9y + 1$$

Homework

Pg 171, #35-48, 61, 63